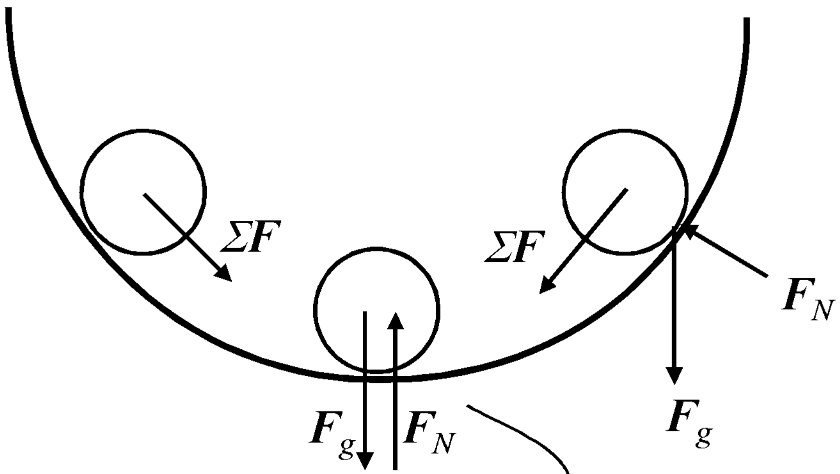
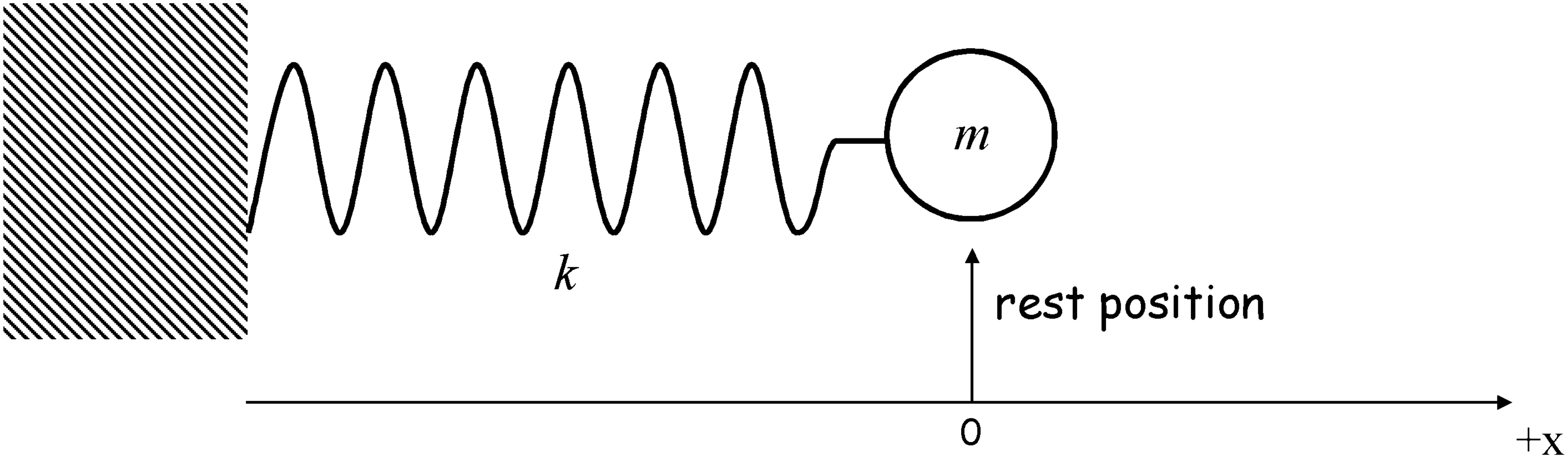
Simple Harmonic Motion (SHM)

Ball in a Bowl:



$$\sum \vec{F} = 0$$

Stable Equilibrium
(*restoring* force, not constant force)



$$\sum \vec{F} = m\vec{a}$$

$$\boxed{-kx = m\ddot{x}}$$

(equation of motion)

Hooke's Law

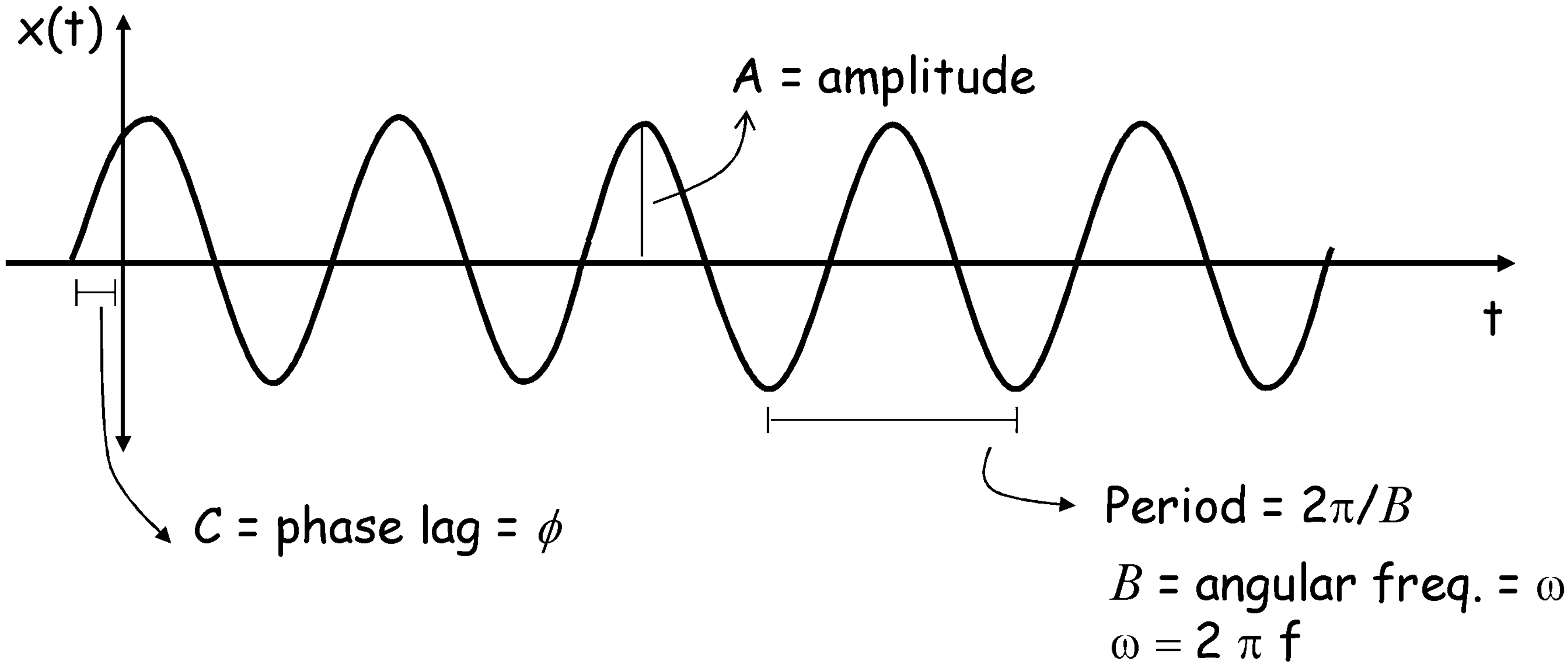
"Guess" a solution: $x(t) = A\sin(Bt + C)$

Does it work?

$$-kA\sin(Bt + C) = -mAB^2 \sin(Bt + C)$$

Yes, if:

$k = mB^2$ ...A and C can take any value



So usually written... $x(t) = A\sin(\omega_o t + \phi)$

$$\omega_o = \sqrt{\frac{k}{m}}$$

A and ϕ from initial conditions.

Example: Mass (m) on spring (k) displaced x = 1 cm at t=0:

$$x(t) = A\sin(\omega_o t + \phi) \qquad \dot{x}(t) = A\omega_o \cos(\omega_o t + \phi)$$

$$x(0) = A\sin(\phi) = 1\text{cm} \qquad \dot{x}(0) = A\omega_o \cos(\phi) = 0$$

$$A\sin\left(\frac{\pi}{2}\right) = 1\text{cm} \longleftarrow \phi = \frac{\pi}{2}$$

$$A = 1\text{cm}$$

$$\boxed{x(t) = (1\text{cm})\sin\left(\sqrt{\frac{k}{m}}t + \frac{\pi}{2}\right)}$$

When a mechanical system is at a point of ***stable equilibrium***, a perturbation will result in a restoring force that drives it back to equilibrium. The resulting equation of motion has the following form:

$$\ddot{x} + \omega_o{}^2 x = 0$$

The solution is an oscillating trajectory at frequency ω_o known as ***Simple Harmonic Motion***.