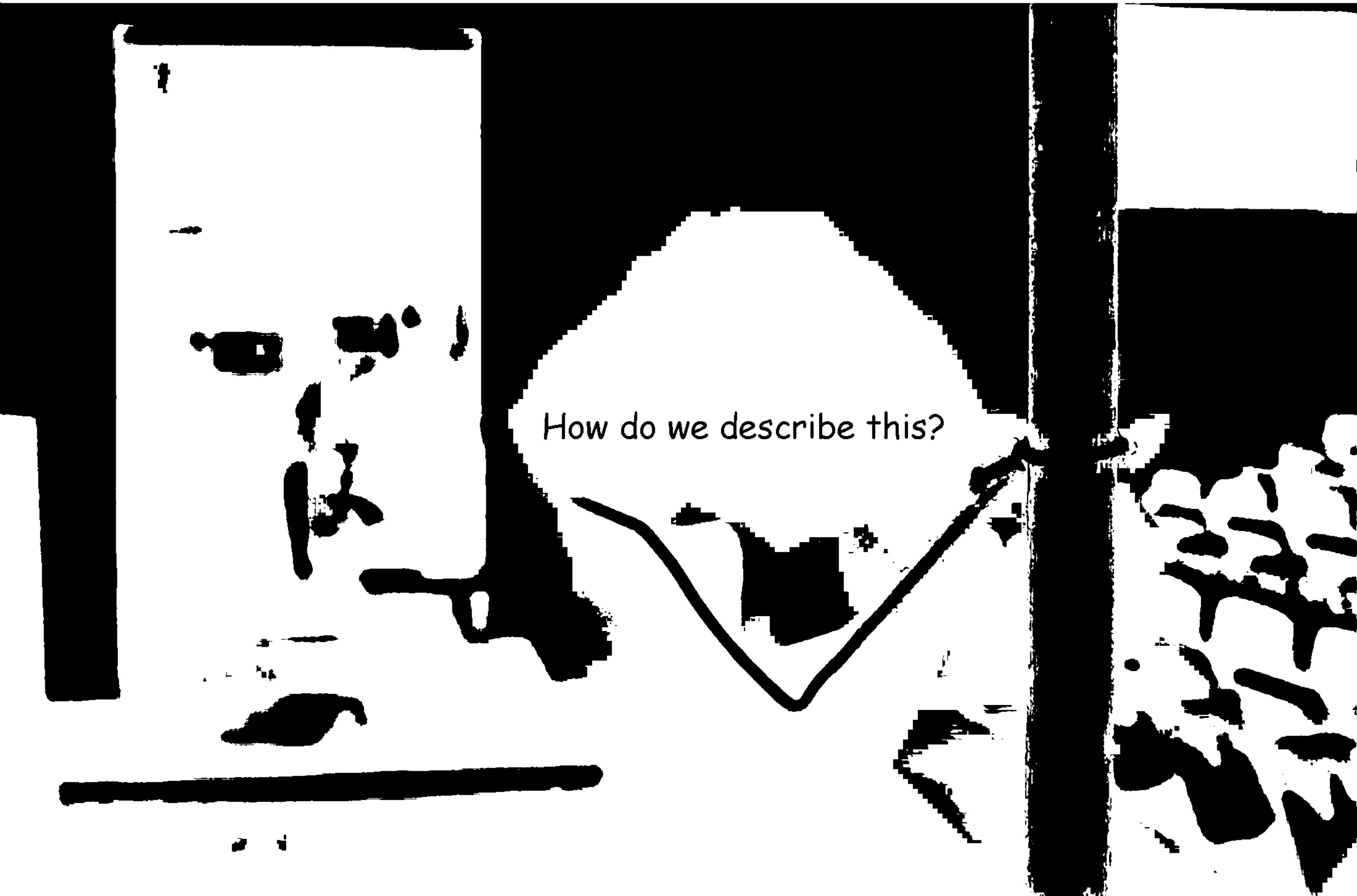
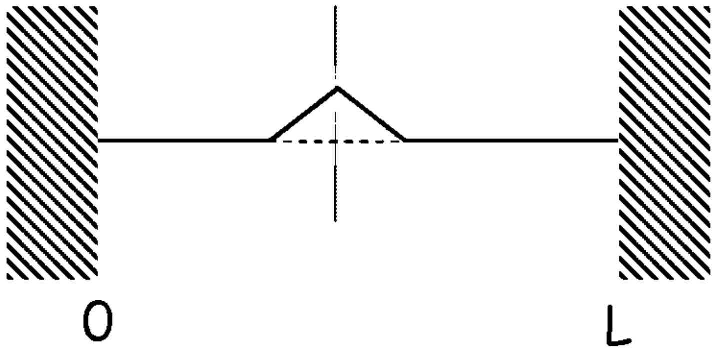
10. Wave Motion

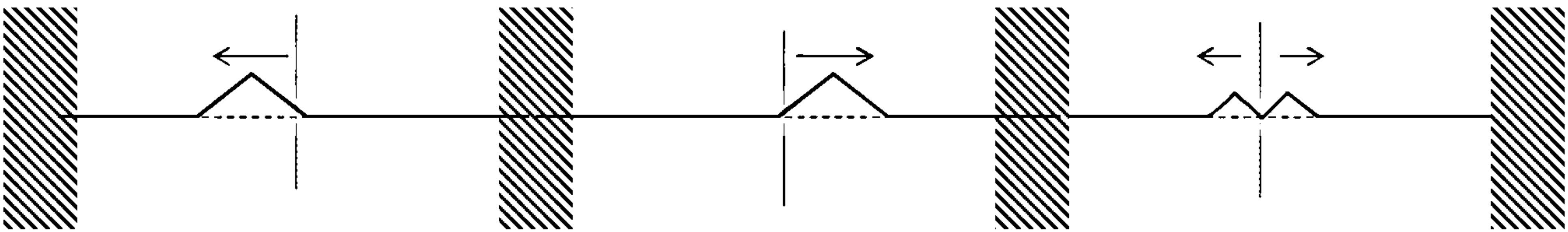


An initial shape…



…so can we just let the normal modes oscillate at ω_n?

No! Because that initial shape can do many things:



To get the answer right, we have to get *all* the boundary conditions right!

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$$

Wave equation needs two spatial and two temporal boundary conditions.

<u>Two spatial boundary conditions</u>

Clamped string: $y(0,t) = 0$

$y(L,t) = 0$

"a certain point in space for all time"

<u>Two temporal boundary conditions</u>

Initial shape: $y(x,0) =$ 

"a certain point in time for all space"

rather than an analogous second point in time (which could work), try this:

Initial velocity: $\dot{y}(x,0) =$ ————

$$y(x,t) = \sum_{n=1}^{\infty} A_n \sin\left(\frac{\omega_n}{v} x + \phi_n\right) \cos(\omega_n t + \delta_n)$$

$$y(0,t) = 0 \longrightarrow \phi_n = 0$$

$$y(L,t) = 0 \longrightarrow \omega_n = \frac{n\pi v}{L}$$

$$y(x,0) = \text{shape} \longrightarrow A_n\text{'s}$$

The $\dot{y}(x,0) =$ velocities tell you how the normal modes move with different phase offset. But don't do it this way!!!

$$y(x,t)=\sum_{n=1}^{\infty}\sin\left(\frac{\omega_n}{v}x\right)\left[B_n\cos(\omega_n t)+A_n\sin(\omega_n t)\right]$$

(spatial boundary conditions) → $\frac{\omega_n}{v}$

$$y(x,0)=\sum_{n=1}^{\infty}\sin\left(\frac{\omega_n}{v}x\right)B_n$$

$$=\sum_{n=1}^{\infty}\sin\left(\frac{n\pi}{L}x\right)B_n$$ French's Fourier series!

$$\dot{y}(x,t)=\sum_{n=1}^{\infty}\sin\left(\frac{\omega_n}{v}x\right)\left[-B_n\omega_n\sin(\omega_n t)+A_n\omega_n\cos(\omega_n t)\right]$$

$$\dot{y}(x,0)=\sum_{n=1}^{\infty}\sin\left(\frac{\omega_n}{v}x\right)\left[A_n\omega_n\right]=0$$

Initially at rest?
Then $A_n=0$