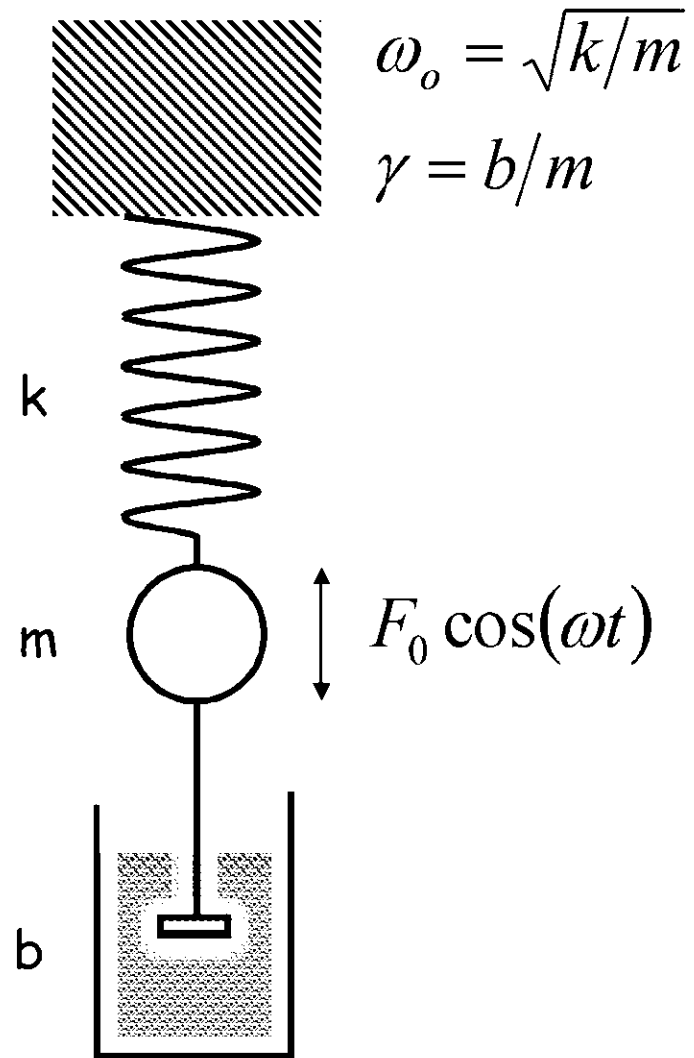


Driven SHM



$$\sum F = m\ddot{x}$$

$$-kx - b\dot{x} + F_0 \cos(\omega t) = m\ddot{x}$$

$$\ddot{x} + \gamma \dot{x} + \omega_o^2 x = \frac{F_0}{m} \cos(\omega t)$$

Driven SHM

steady state: $z_{ss}(t) = A_{ss} e^{j(\omega t - \delta_{ss})}$ (*Resistance is futile!!!*)

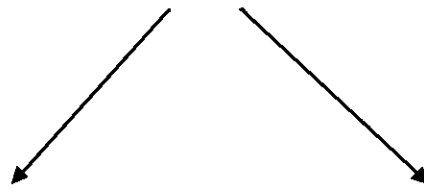
ω = steady state freq. = *the drive frequency*

A_{ss} = steady state amplitude

δ_{ss} = steady state phase lag between drive and motion

$$j^2 \omega^2 A_{ss} e^{j(\omega t - \delta_{ss})} + \gamma j \omega A_{ss} e^{j(\omega t - \delta_{ss})} + \omega_o^2 A_{ss} e^{j(\omega t - \delta_{ss})} = \frac{F_0}{m} e^{j\omega t}$$

$$-\omega^2 A_{ss} + \gamma j \omega A_{ss} + \omega_o^2 A_{ss} = \frac{F_0}{m} e^{j\delta_{ss}}$$



Real

$$-\omega^2 A_{ss} + \omega_o^2 A_{ss} = \frac{F_0}{m} \cos(\delta_{ss})$$

Imaginary

$$\gamma j \omega A_{ss} = j \frac{F_0}{m} \sin(\delta_{ss})$$

ratio:

$$\frac{\sin(\delta_{ss})}{\cos(\delta_{ss})} = \frac{\gamma \omega A_{ss}}{-\omega^2 A + \omega_o^2 A_{ss}}$$

$$\tan(\delta_{ss}) = \frac{\gamma \omega}{\omega_o^2 - \omega^2}$$

(steady state)

$$\sin^2(\delta_{ss}) + \cos^2(\delta_{ss}) = 1$$

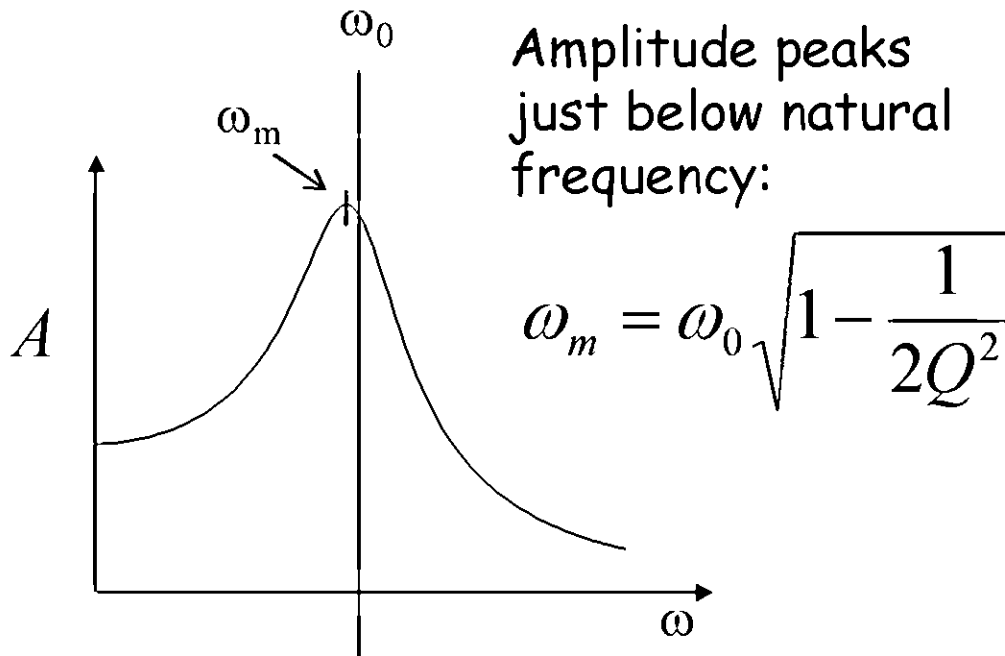
$$\left(\frac{m\gamma\omega A_{ss}}{F_0}\right)^2 + \left(\frac{m}{F_0}\right)^2 \left(A_{ss}\omega_0^2 - A_{ss}\omega^2\right)^2 = 1$$

$$\left(\frac{A_{ss}m}{F_0}\right)^2 \left(\left(\omega_0^2 - \omega^2\right)^2 + (\gamma\omega)^2\right) = 1$$

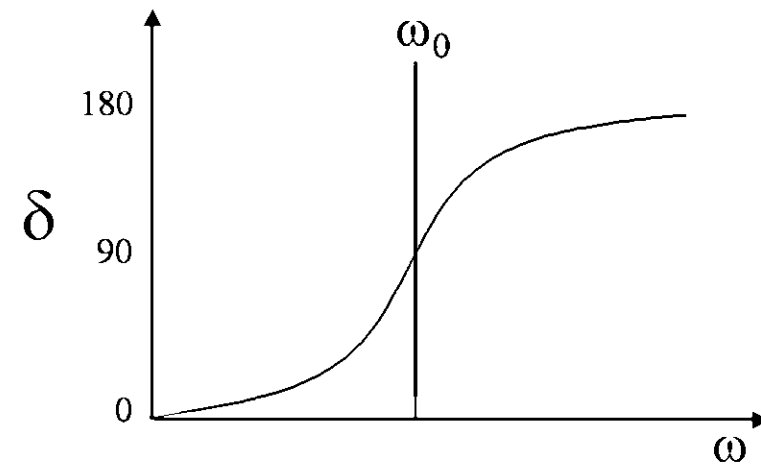
$$A_{ss} = \frac{F_0/m}{\sqrt{\left(\omega_0^2 - \omega^2\right)^2 + (\gamma\omega)^2}} \quad \text{(steady state)}$$

steady state solution:

$$x_{ss}(t) = \frac{F_0/m}{\sqrt{(\omega_0^2 - \omega^2)^2 + (\gamma\omega)^2}} \cos\left(\omega t - \tan^{-1}\left(\frac{\gamma\omega}{\omega_0^2 - \omega^2}\right)\right)$$



Phase lag through 90 deg exactly at natural frequency.



Steady state: A_{ss} and δ_{ss} are not adjustable parameters, they are set by the properties of the system just like ω_0 !!!

For the missing part, add our result for the damped oscillator:

$$\ddot{x} + \gamma \dot{x} + \omega_o^2 x = 0$$

$$x_{tr}(t) = Ae^{-\gamma t/2} \cos\left(\left(\omega_o^2 - \gamma^2/4\right)^{1/2} t + \phi\right) \quad (\text{last lecture})$$

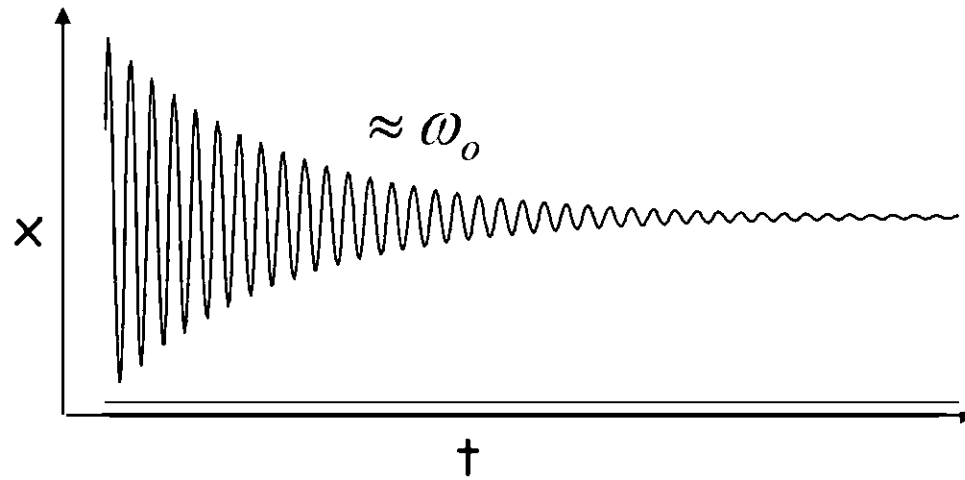
$$\ddot{x}_{ss} + \gamma \dot{x}_{ss} + \omega_o^2 x_{ss} = \frac{F_0}{m} \cos(\omega t - \delta)$$

$$+ \quad \ddot{x}_{tr} + \gamma \dot{x}_{tr} + \omega_o^2 x_{tr} = 0$$

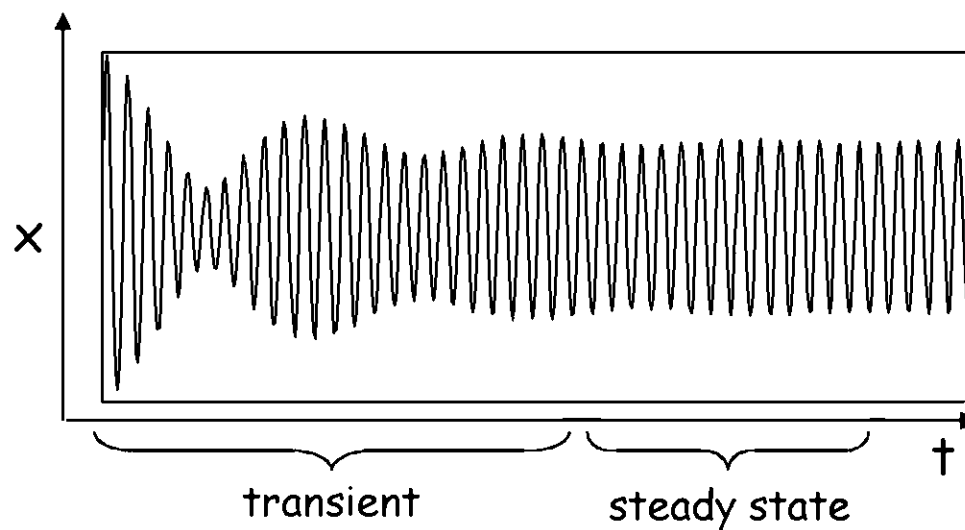
$$\left(\ddot{x}_{ss} + \ddot{x}_{tr}\right) + \gamma\left(\dot{x}_{ss} + \dot{x}_{tr}\right) + \omega_o^2\left(x_{ss} + x_{tr}\right) = \frac{F_0}{m} \cos(\omega t - \delta)$$

$$x(t) = x_{tr}(t) + x_{ss}(t)$$

Damped

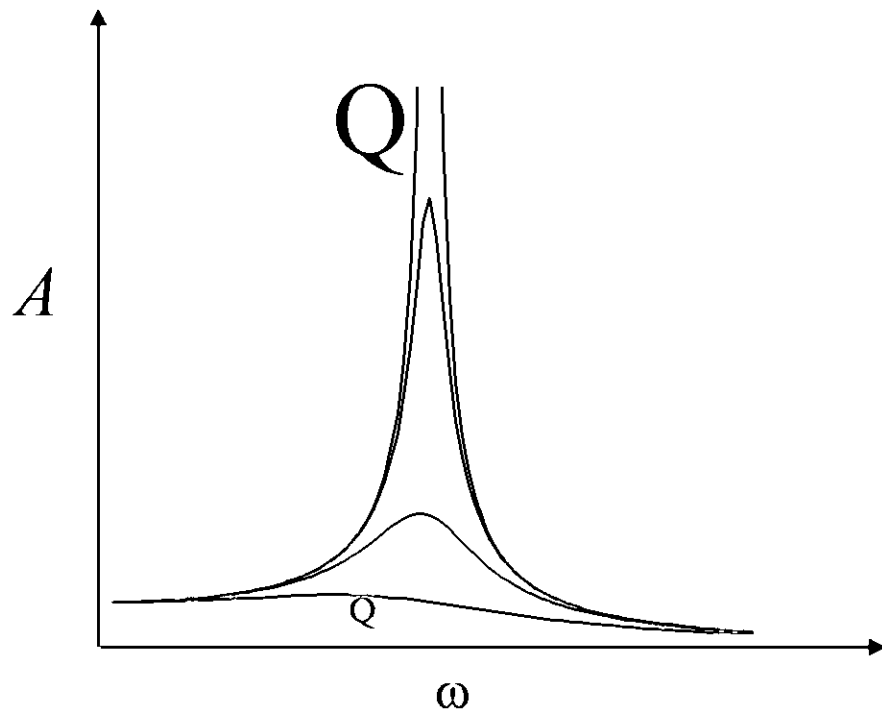


Driven & Damped



$$\sqrt{\omega_0^2 - \gamma^2/4} \text{ and } \omega \quad \omega$$

High Q = "sharp" resonance



$$Q \equiv \frac{\omega_0}{\gamma}$$

$$Q \approx \frac{\omega_0}{\Delta\omega_{FWHM}}$$

(for high Q only)

A simple harmonic oscillator driven by a sinusoidal force oscillates at the frequency of the force once the transient motion at the natural frequency decays. The steady state amplitude goes through a resonance near the natural frequency.