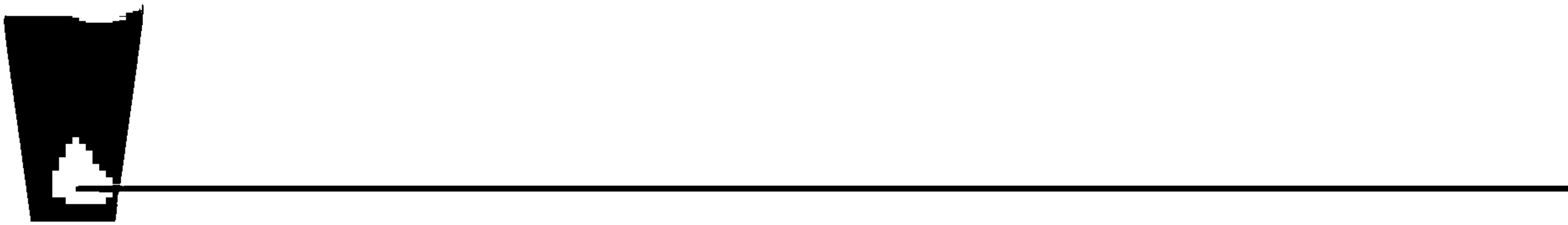
Reflection and Transmission

What force must be applied at end of a string to launch a pulse?



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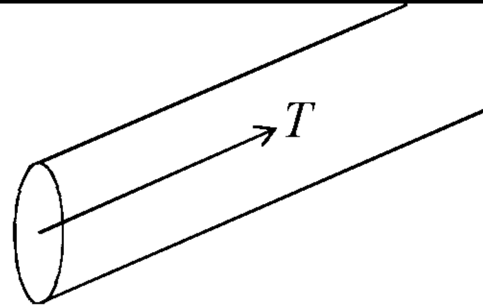
THERE ARE TWO FORCES TO CONSIDER !!!!!!

Force on a string segment Δx:
(from when we derived the wave equation)

$$\sum F_y \approx T\frac{\partial y}{\partial x}(x+\Delta x) - T\frac{\partial y}{\partial x}(x) = \mu(\Delta x)a$$

As Δx goes to zero, F_y and mass go to zero, but acceleration of the string segment remains finite.

It had better – we need a force to move the string to get a wave equation !



Vertical force at the string end:

$$F_y = T\sin(\theta)$$

$$F_y \approx T\left(\frac{\partial y}{\partial x}\right)$$ (Tension force at end)

<u>The Impedance Creed</u>

The string end is an interface.
It has no width. It has no mass.
There is no limit to apply.
It cannot have a net force applied to it.
A net force would be equivalent to an infinite stress.
A net force at the end would rip the string.
The driver must therefore balance this force.

$$F_{drive} = -T\left(\frac{\partial y}{\partial x}\right)$$

If $y = f(x\text{-}vt)$:

$$F_{drive} = -T\left(\frac{\partial y}{\partial x - vt}\frac{\partial x - vt}{\partial x}\right)_{x=0}$$

$$F_{drive} = -T\left(\frac{\partial y}{\partial x - vt}\right)_{x=0}$$

$$F_{drive} = -T\left(\frac{\partial y}{\partial t}\frac{\partial t}{\partial x - vt}\right)_{x=0}$$

$$F_{drive} = \frac{T}{v}\left(\frac{\partial y}{\partial t}\right)_{x=0}$$

$$F_{drive} = \sqrt{T\mu}\left(\frac{\partial y}{\partial t}\right)_{x=0}$$

Force needed is proportional to the *transverse velocity*, like a damping force!

Constant of proportionality is the characteristic impedance:

$$Z = \sqrt{T\mu}$$

The change in impedance at an interface determines the amounts of reflection and transmission.

$$Z_1 = \sqrt{T\mu_1} \qquad\qquad Z_2 = \sqrt{T\mu_2}$$

$$f_1(x - vt) \longrightarrow \qquad\qquad f_2(x - vt) \longrightarrow$$

$$g_1(x + vt) \longleftarrow$$

The string must be continuous at the boundary:

$$f_1(t) + g_1(t) = f_2(t)$$

The force must balance at the boundary:

$$F_y = -T\frac{\partial f_1}{\partial x} - T\frac{\partial g_1}{\partial x}$$

$$F_y = -T\frac{\partial f_2}{\partial x}$$

$$F_y = Z_1\left(\frac{\partial f_1}{\partial t} - \frac{\partial g_1}{\partial t}\right)$$

$$F_y = Z_2\left(\frac{\partial f_2}{\partial t}\right)$$

$$Z_1\left(\frac{\partial f_1}{\partial t} - \frac{\partial g_1}{\partial t}\right) = Z_2\left(\frac{\partial f_2}{\partial t}\right)$$

Integrate with respect to time:

$$Z_1 f_1(t) - Z_1 g_1(t) = Z_2 f_2(t)$$

$$Z_1 f_1(t) - Z_1 g_1(t) = Z_2 f_1(t) + Z_2 g_1(t)$$

$$Z_1 g_1(t) + Z_2 g_1(t) = Z_1 f_1(t) - Z_2 f_1(t)$$

$$r = \frac{g_1(t)}{f_1(t)} = \frac{Z_1 - Z_2}{Z_1 + Z_2}$$

reflection coefficient

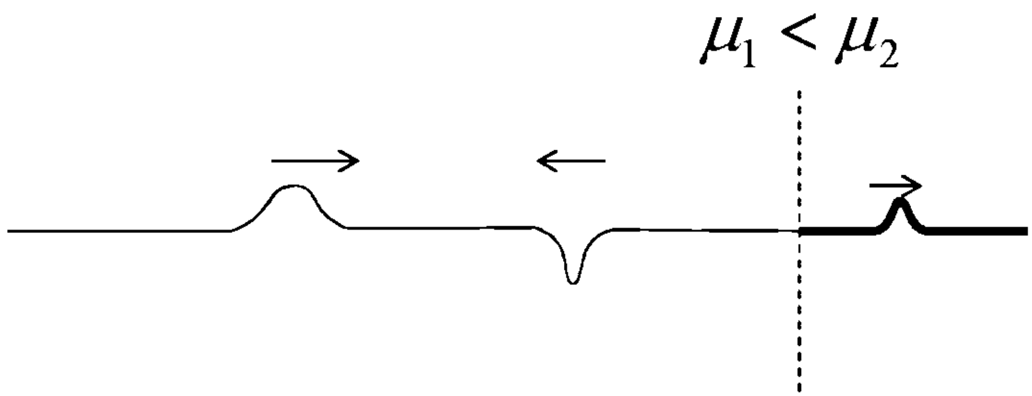
$$Z_1 f_1(t) - Z_1 f_2(t) + Z_1 f_1(t) = Z_2 f_2(t)$$

$$-Z_1 f_2(t) - Z_2 f_2(t) + = -2Z_1 f_1(t)$$

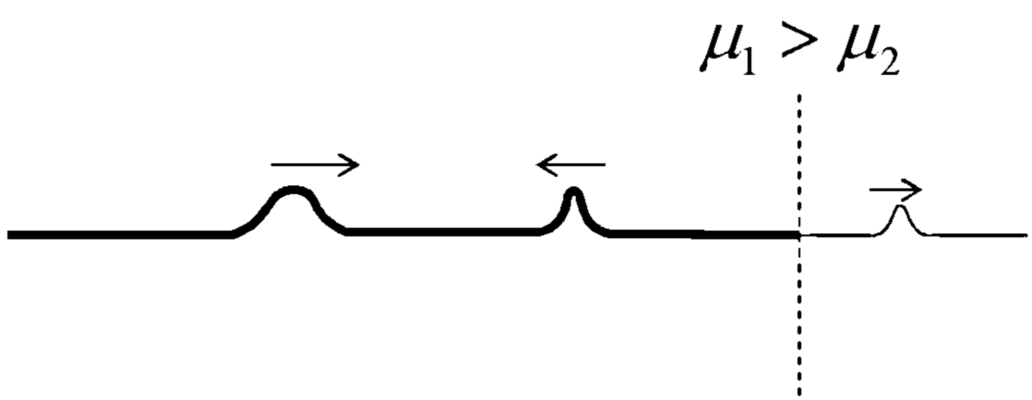
$$t = \frac{f_2(t)}{f_1(t)} = \frac{2Z_1}{Z_1 + Z_2}$$

transmission coefficient

Into more dense medium



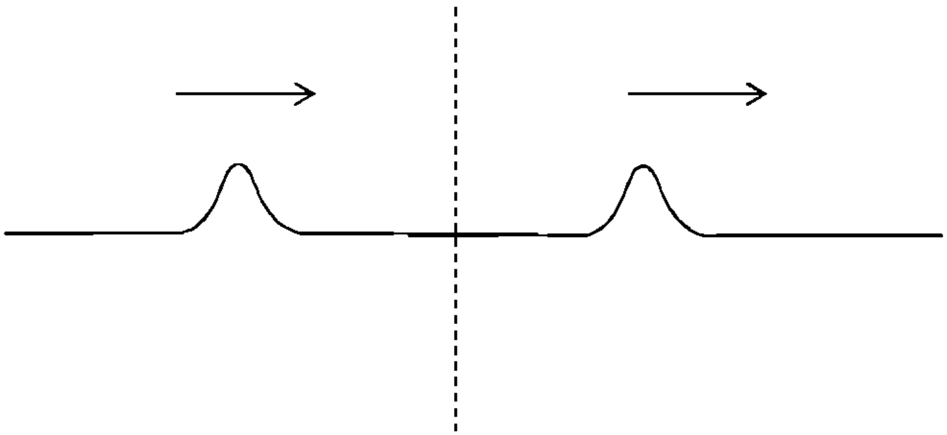
Into less dense medium



Impedance Matching: No Reflection!

$$Z_1 = Z_2$$

$$\sqrt{T_1 \mu_1} = \sqrt{T_1 \mu_1}$$



(not really an interface)

$$\sqrt{T_1 \mu_1} = \sqrt{T_2 \mu_2}$$